Lamellar (non-Curl) vector field

If a vector field $\vec{A}$ can be expressed as the gradient of a scalar field, then $\vec{A}$ is called a lamellar vector field.

A pure electric field $\vec{E}$ can be expressed as gradient of a scalar potential ϕ, i.e.,

$\vec{E} = -\nabla\phi$ thus $\vec{E}$ is a lamellar vector field.

The Divergence of a vector function:

The divergence of a vector field at any point is defined as the amount of flux diverging through the surface enclosing unit volume.

mathematically

$$\text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence of a vector function $\vec{A}$ is itself a scalar $\vec{\nabla}\cdot\vec{A}$ (divergence of a scalar quantity is meaningless)

Let there be a vector field $\vec{A}$ in certain region and v be the infinitesimal volume element enclosed by an infinitely small closed surface $\vec{S}$ surrounding a point $P(x, y, z)$.

$$\text{div}\,\vec{A} = \lim_{v\to 0} \frac{\iint_S \vec{A}\cdot d\vec{S}}{v}$$

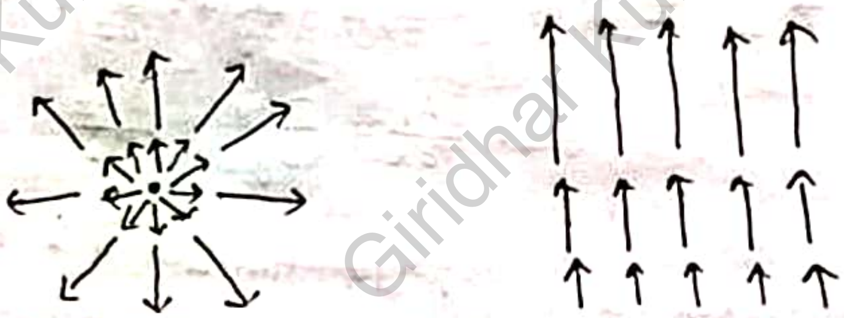
Physical significance

If $\vec{A}$ represents the velocity of a moving fluid at any point P, then div $\vec{A}$ gives the rate at which the fluid is diverging per unit volume from the point P.

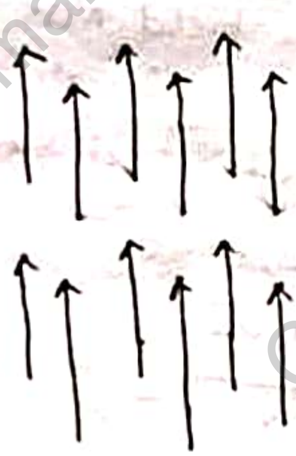
If div $\vec{A} > 0$ at any point P, then either the fluid is expanding or the point P is a source & if div $\vec{A} < 0$, then either the fluid is contracting or the point P is a sink.

If div $\vec{A} = 0$, then flux of $\vec{A}$ entering any element of space is exactly balanced by the flux leaving it.

A vector $\vec{A}$ that satisfies the condition div $\vec{A} = 0$, is called a solenoidal vector.



Non-zero divergence.



zero divergence.